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MODELLING VOLATILITY: SYMMETRIC OR ASYMMETRIC GARCH MODELS?

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Abstract

In this paper, we estimate GARCH, EGARCH, and GJR-GARCH models assuming normal and heavy-tailed distribution (i.e., GED). Results suggest that when the heavy-tailed distribution is considered, the persistence has found to be reduced in all the cases. Findings also reveal that positive shocks are more common than the negative shocks in this market.

1. Introduction

Although generalized autoregressive conditional heteroscedastic (GARCH) models have a long and comprehensive history over the years, they are not free of limitations. Black [1], for example, documents that stock returns are negatively correlated to changes in returns volatility implying that volatility tends to rise in response to bad news and fall in response to good news. GARCH models, on the other hand, assume that only magnitude and not the positivity or negativity unanticipated excess return determines the conditional variance. This suggests that a model in

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which the conditional variance responds asymmetrically to positive and negative residuals might be preferable for asset pricing applications. Another crucial limitation of a GARCH model is that the non-negativity constraints on its parameters are imposed to ensure the positivity of the conditional variance. Such constraints can create difficulties in estimating GARCH models.

Nelson [7], however, introduces the exponential GARCH or EGARCH model to present potential improvements over the conventional GARCH models. The EGARCH model earns its popularity due to the fact that it presents the asymmetric response of volatility to positive and negative returns. This model is also commonly used as it shares some of its properties with GARCH model. Glosten et al. [6] also propose another asymmetric GARCH model, popularly known as GJR-GARCH model, to deal with the limitation of symmetric GARCH models.

The objective of this paper is twofold. First, we observe and compare the properties of symmetric and asymmetric GARCH models by analyzing the U.S.-Japan daily exchange rate series. To serve this purpose, we estimate GARCH, EGARCH, and GJR-GARCH models assuming normal and heavy-tailed distribution (i.e., GED). Our second objective is to verify whether incorporating asymmetric response of volatility to and negative shocks changes the conclusions obtained from the symmetric GARCH models. The rest of the paper is organized in the following way. Section 2 discusses the symmetric and asymmetric GARCH models. Section 3 summarizes the data. The results are discussed in Section 4 and Section 5 concludes the paper.

2. Methodology

2.1. GARCH models

The basic model of representing non-correlated series with excess kurtosis and autocorrelated squares, proposed by Engle [3], is given by

$$\varepsilon_t = z_t \sigma_t,$$

where z_t is an i.i.d process with mean zero and variance 1 and σ_t is the volatility that evolves over time. The volatility, σ_t^2, in the basic ARCH (1) model is defined as

$$\sigma_t^2 = w + \alpha\varepsilon_{t-1}^2,$$

where $w > 0$ and $\alpha \geq 0$ for σ_t^2 to be positive.

The ARCH (1) model can easily be extended to the ARCH (q) model

$$\sigma_t^2 = w + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2.$$

However, early applications of ARCH models needed many lags to adequately represent the dynamic evolution of the conditional variances. In some applications, q could be even 50. To avoid computational problems when estimating such a large number of parameters, the parameters were restricted in an *ad hoc* manner. For example, Engle [4] assumes that $\alpha_i = \dfrac{\alpha(q+1-i)}{q(q+1)}$. Later, Bollerslev [2] implements the same kind of restriction used to approximate the infinite polynomial of the Wald representation by the ratio of two finite polynomials, usually of very low orders. As a result, he proposed the GARCH (p, q) model given by

$$\sigma_t^2 = w + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2.$$

Then the GARCH (1, 1) model is simply given by

$$\sigma_t^2 = w + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2,$$

where $w > 0$, $\alpha \geq 0$, and $\beta \geq 0$ for σ_t^2 to be positive.

2.2. EGARCH models

The standard GARCH model has a number of potential pitfalls. Such models cannot take into consideration asymmetry, leverage effects, and coefficient restrictions. Nelson [7] proposes the exponential GARCH or EGARCH model to resolve these limitations. Unlike the standard GARCH model, the EGARCH model can capture size effects as well as sign effects of shocks. The variance equation of EGARCH model is given as follows:

$$\ln(\sigma_t^2) = w + \sum_{i=1}^{q} \alpha_i \frac{|\varepsilon_{t-i}| + \delta_i \varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{p} \beta_j \ln(\sigma_{t-j}^2).$$

When ε_{t-1} is positive or there is good news, the total effect ε_{t-i} of is $(1 + \delta_i)|\varepsilon_{t-i}|$. In contrast, when ε_{t-i} is negative or there is bad news, the total impact of ε_{t-i} is $(1 - \delta_i)|\varepsilon_{t-i}|$. Besides, this model captures the leverage effect, which exhibits the negative association between lagged stock returns and contemporaneous volatility. The presence of leverage effects can be tested by the hypothesis that $\delta < 0$. If $\delta \neq 0$, then the impact is asymmetric.

2.3. GJR-GARCH models

Glosten et al. [6] suggest the GJR-GARCH model as an alternative method to the EGARCH model. Like the EGARCH model, the GJR-GARCH model has also achieved a good empirical record in the literature. The variance of this model can be written as

$$\sigma_t^2 = w + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \delta S_{t-i}^{-} \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2,$$

where S_{t-i}^{-} is a dummy variable which takes the value 1 if ε_{t-i} is negative and 0 otherwise. The formula expresses the impact of ε_{t-i}^2 on the conditional variance σ_t^2. The above model also confirms that bad news $(\varepsilon_t < 0)$ and good news $(\varepsilon_t > 0)$ might have different effects on

conditional variance. If the leverage effect exists, δ is expected to be positive. The leverage effect is observed as the impulse $(\alpha + \delta)$ of negative shocks, which is larger than the impulse (α) of positive shocks. In this model, good news and bad news have different effects on the conditional variance: good news has an impact of α, while bad news has an impact of $(\alpha + \delta)$. For $\delta > 0$, the leverage effect exists.

2.4. Distribution hypotheses

The probability distribution of asset returns often exhibits fatter tails than the standard normal distribution. The existence of heavy-tailedness is probably due to a volatility clustering in stock markets. In addition, another source for heavy-tailedness seems to be the sudden changes in stock returns. An excess kurtosis also might be originated from fat tailedness. Moreover, in practice, the returns are typically negatively skewed. In order to capture this phenomenon (e.g., heavy-tailedness), the GED distribution is also considered in our analysis.

2.5. Tests for asymmetries in volatility

Engle and Ng [5] have proposed a set of tests, known as sign and size bias tests, in order to determine whether an asymmetric model is required for a given series, or whether the symmetric GARCH model can be deemed adequate. Such tests are usually applied to the residuals of a GARCH fit to the returns data. The test for sign bias is based on the significance or otherwise of b_1 in the following regression:

$$z_{i,t}^2 = b_0 + b_1 D_{i,t-1}^- + v_t,$$

where $D_{i,t-1}^-$ is a dummy variable which takes the value 1 if ε_{t-i} is negative and 0 otherwise and v_t is an i.i.d. error term.

It could also be the case that the magnitude or size of the shock will affect whether the response of volatility to shocks is symmetric or not. In this case, a negative size bias test would be conducted. Negative size bias

is argued to be present if b_1 is statistically significant in the regression given below:

$$z_{i,t}^2 = b_0 + b_1 D_{i,t-1}^{-} \varepsilon_{i,t-1} + v_t.$$

Similarly, positive size bias is said to be present if b_1 is statistically significant in the regression given below:

$$z_{i,t}^2 = b_0 + b_1 (1 - D_{i,t-1}^{-}) \varepsilon_{i,t-1} + v_t.$$

2.6. Likelihood ratio tests

Likelihood ratio (LR) tests involve estimation under the null hypothesis and under the alternative so that two models are estimated: An unrestricted model and a model where the restrictions have been imposed. The maximized values of the log-likelihood function (LLF) for the restricted and unrestricted cases are compared. Suppose that the unconstrained model has been estimated and that a given maximized value of the LLF, denoted by L_u, has been achieved. Suppose also that the model has been estimated imposing the constraint(s) and a new value of the LLF obtained, denoted by L_r. The LR test statistic asymptotically follows a Chi-squared distribution and is given by

$$LR = -2(L_r - L_u) \sim \chi_m^2,$$

where m denotes the number of restrictions.

3. Data and their Properties

In this paper, we employ U.S.-Japan daily exchange rate series which ranges from January 1, 2000 to January 31, 2012. We compute the return series as follows:

$$r_t = 100 * \ln \frac{P_t}{P_{t-1}},$$

where P_t indicates the observed daily price at time t and r_t is the corresponding daily return. Table 1 reports the main empirical properties

of the data set under consideration. It is evident from Table 1 that the mean value for the stock return is positive indicating that positive changes in stock price indices are more dominant than negative changes. The skewness and kurtosis coefficients report that stock market returns are leptokurtic and positively skewed with respect to the normal distribution (skewness = 0, kurtosis = 3).

Table 1. Summary statistics of the return series

Number of Observations	Mean	STDEV	Skewness	Kurtosis
3050	0.00969	0.66400	0.355	3.531

4. Results and Discussions

4.1. Estimation of models and volatility persistence

This section indicates and illustrates the results obtained by fitting symmetric and asymmetric GARCH models to the return series considered in this study. Table 2 reports the estimates obtained by GARCH, EGARCH, and GJR-GARCH models. It is interesting to note that the volatility persistence significantly decreased when heavy-tailed conditional density is considered. Thus, the heavy-tailed distributions play an important role in the reduction of persistence. In sum, distribution hypothesis seems to play a significant role in the estimation of persistence.

Table 2. Estimated models for the daily returns

	GARCH		EGARCH		GJR-GARCH	
	Normal	GED	Normal	GED	Normal	GED
	$\hat{w}$ = 0.007	$\hat{w}$ = 0.006	$\hat{w}$ = 0.092	$\hat{w}$ = 0.084	$\hat{w}$ = 0.007	$\hat{w}$ = 0.007
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	$\hat{\alpha}$ = 0.033	$\hat{\alpha}$ = 0.031	$\hat{\alpha}$ = 0.094	$\hat{\alpha}$ = 0.089	$\hat{\alpha}$ = 0.047	$\hat{\alpha}$ = 0.044
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	$\hat{\beta}$ = 0.950	$\hat{\beta}$ = 0.951	$\hat{\beta}$ = 0.977	$\hat{\beta}$ = 0.981	$\hat{\beta}$ = 0.954	$\hat{\beta}$ = 0.952
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
			$\hat{\delta}$ = 0.042	$\hat{\delta}$ = 0.034	$\hat{\delta}$ = –.038	$\hat{\delta}$ = –.032
			(0.000)	(0.000)	(0.000)	(0.000)
Persistence	0.983	0.982	0.977	0.981	0.981	0.980
Log-likelihood	– 2974.47	– 2895.15	– 2956.84	– 2886.18	– 2965.06	– 2890

Notes: The persistence is calculated as $\hat{\alpha} + \hat{\beta}$ for GARCH model, $\hat{\beta}$ for EGARCH model, and $\hat{\alpha} + \hat{\delta}/2 + \hat{\beta}$ for GJR-GARCH model. Values in parentheses denote the *p*-values.

4.2. Asymmetric and leverage effects

The asymmetric and leverage effects can be examined by the nonlinear asymmetric variance specifications, EGARCH and GJR-GARCH, under different distribution assumptions. The coefficient is found statistically significant in all cases indicating that asymmetry does exist. However, the sign of is positive in EGARCH model and negative in GJR-GARCH model under all distributions implying that there exist no leverage effects. In addition, good news seems to have more impact on volatility than the bad news. As Table 3 presents, both EGARCH and GJR-GARCH models reveal that good news has an impact on volatility more than bad news under different distribution assumptions. In the GJR-GARCH model assuming GED, for instance, the effect of good news on conditional volatility is 3.67 times more than bad news.

Table 3. The magnitude of news impact on volatility

	EGARCH		GJR-GARCH	
	Normal	GED	Normal	GED
Bad news	0.958	0.966	0.009	0.012
Good news	1.042	1.034	0.047	0.044

Notes: The asymmetry is calculated as $\frac{|-1+\hat{\delta}|}{1+\hat{\delta}}$ for EGARCH model and $\frac{\hat{\alpha}+\hat{\delta}}{\hat{\alpha}}$ for GJR-GARCH model.

4.3. Test of asymmetries

Table 4 reports the results for testing asymmetries in volatility. Findings show that the sign bias test statistics are significant for the asymmetric models. The two size bias test statistics are also highly significant with positive size bias test statistics having higher values in all the cases. These results indicate a size effect of news, which is stronger for good news than for bad news.

Table 4. Tests of asymmetries

	GARCH		EGARCH		GJR-GARCH	
	Normal	GED	Normal	GED	Normal	GED
Sign bias	– 1.61	– 1.53	– 1.98	– 1.99	– 2.03	– 2.09
Negative size bias	– 23.79	– 20.48	– 24.26	– 23.68	– 21.99	– 23.77
Positive size bias	38.03	41.62	37.78	38.14	39.90	38.37

4.4. Likelihood ratio tests

Table 2 shows the maximized values of different GARCH models under different distributional assumptions. For example, a maximized LLF of 2890 is achieved for the GJR-GARCH model assuming GED. Now, to test $H0 : \delta = 0$, the model is estimated imposing the restriction and the maximized LLF falls to – 2895.15 as shown in Table 2. The test statistic is given by LR = – 2(– 2895.15 – 2890) = 10.30. Since the test follows a $\chi_1^2 = 3.84$ at 5 percent level of significance, the null is rejected. Similar inference can be drawn when GJR-GARCH model is estimated assuming normality.

5. Conclusion

In this paper, we study and compare the properties of symmetric and asymmetric GARCH models by analyzing the U.S.-Japan daily exchange rate series. In doing so, we estimate GARCH, EGARCH, and GJR-GARCH models assuming normal and heavy-tailed distribution (i.e., GED). Our analysis reveals that when the heavy-tailed distribution is considered, the persistence is found to be reduced in each of the cases under study. We also make an attempt to verify whether incorporating asymmetric response of volatility to positive and negative shocks changes the conclusions obtained from the symmetric GARCH models. Findings indicate that positive shocks are more common than the negative shocks in this return series. However, tests for asymmetries in volatility indicate a size effect of news, which is stronger for good news than for bad news.

References

[1] F. Black, Studies in stock price volatility changes of the nominal excess return on stocks, Proceedings of the 1976 Business Meeting of the Business and Economics Statistics Section, American Statistical Association (1976), 177-181.

[2] T. Bollerslev, Generalized autoregressive conditional heteroskedasticity, Journal of Econometrics 73 (1986), 151-184.

[3] R. F. Engle, Autoregressive conditional heteroskedasticity with estimates of the variance of UK inflation, Econometrica 50 (1982), 987-1007.

[4] R. F. Engle, Estimates of the variance of U. S. inflation based on the ARCH model, Journal of Money, Credit and Banking 15 (1983), 286-301.

[5] R. F. Engle and N. Ng, Measuring and testing the impact of news on volatility, Journal of Finance 48 (1993), 1749-1778.

[6] L. R. Glosten, R. Jagannathan and D. E. Runkle, On the relation between the expected value and the volatility of the nominal excess return on stocks, The Journal of Finance 48 (1993), 1779-1801.

[7] D. B. Nelson, Conditional heteroskedasticity in asset returns: A new approach, Econometrica 59 (1991), 347-370.

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